Using Optimal Golden-Fractal Geometrical Shapes to Generate Sustainable and Healthy Interior Environment

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Abstract:

Nowadays, the importance of the shapes existence in interior design and furniture is not only for aesthetic interest, but for the functional, light weight structural and environmental adaptability, comfort of the human wellbeing and sustainability. For long time, the interior designer was inspired by Euclidean geometrical shapes (e.g. triangle, square, and polyhedral), that made the interior design and furniture rigid, non- adaptive and inefficient design with waste of materials. The evolution in computerized design technologies have permitted designers to overcome the limits imposed by Euclidean regular geometry as well as Euclidean shapes and replaced it with the Fractal geometry (non-Euclidean geometry). The fractal geometry is applied to create new kinds of irregular shapes of optimum structures and thus provides high level of force with minimum used mass through the design systems.

The researcher proposed three new simple models to obtain golden fractal “light-weight, high-strength” structure design, that can be used in interior design and furniture applications. **The first model**: two new golden fractal self -symmetry binary trees with symmetry angle =45° and angle=90°, based on using the golden rectangle shape. **The second model:** new hybrid golden fractal shape, consist of two golden pentagon joined together with golden triangles, based on using the golden ratio relation $\varphi^{n}=\varphi^{n-1}+\varphi^{n-2}$. **The third model:** an optimal golden fractal shapes based on using a mathematical prediction and geometrical analysis technique to obtain a finite mid-range interval (1,5+ϵ, 1,5-ϵ) where ϵ is a positive sufficiently small value for optimal fractal dimension . These three models used fractal geometry hierarchical self-similar arranging property and a four-steps computer IFS iterative code to maintain high strength and light weight optimal golden fractal structure design. The researcher used the golden geometrical shape as an initial shape in the iterative process because it emits positive energy, create balance in the interior environment and give wellbeing to human health.

Some of the advantages of using optimal golden fractal “light-weight, high-strength” structure design in interior design and furniture are: being of light weight, therefore, minimize the amount of used materials, the space arrangement becomes quick and easy, thus, changing the static interior space into dynamic. It encourages adaptability and flexibility in the environment. Since, the structure is being of high strength, it can stand the continuous and prolonged duration of functional usage. It brings aesthetical to the space, emits positive energy, and creates balance in the interior environment. It generates sense of wellbeing and comfort as it damps automatically the human body response to stress. Therefore, we have to embed and manipulate optimal golden fractal in interior design and furniture compositions or at least mimic the mechanism of optimal golden fractal to convey emotion and sensation to occupants.

Keywords:

Fractal geometric shape, Iterated function system IFS, Sustainability, Fractal dimension, Hierarchical Self-similarity, Contract transformation, Computational design, Optimal golden fractal structure design.

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Introduction:

Fractal geometry is a modern geometry that departs radically from traditional Euclidean geometry. The acceptance of the word fractal was dated in 1975 when the French mathematician “Mandelbrot” presented his list of publications between 1951 and 1975(1),(2). Fractal geometry is a new area of research which provides an extremely irregular powerful tool in the field of computational design and applications. Increasing adoption of computational techniques in designing indicates the presence of a significant potential in making a considerably greater use of fractal geometry in interior and furniture design. It appears that, although fractals are well understood and extensively analyzed from a mathematical perspective yet their use in interior design and furniture has been very limited. A gap exists between the mathematics of fractal geometry and its applications in interior design and furniture. The Contemporary interior design and furniture inspired from natural fractals have good effects related to functionality, aesthetically, adaptability, sustainability, human health, and comfort, as well as creating balance in interior environment. So, to achieve these advantages it is necessary to inspire from the geometrical fractal especially after the progress in computer technology. The field of applications of structural fractal shapes in interior design and furniture are useful, but rare, as compared to other fields of design. This observation gives us the idea of applying fractal geometry with hierarchical self-similar repetition in interior design and furniture.

Research problem:

1-The world faces a big challenge with increasing the population and decreasing in the material resources. Therefore, there is a necessity to create a new design vision to satisfy functionality and aesthetically with sustainable and healthy environment.

2- Interior designers have not adopted fractal shapes application in their interior and furniture structure designs despite its good properties and advantages and paid more attention to Euclidean geometric system for its simplicity.

Research objectives

1- Exploring the advantages of fractal structural design properties, so it can be applied in the field of interior design and furniture.

2- Bridging the gap between the mathematics of fractal geometry with its applications using the computer in interior design and furniture and the interior designers to be able to apply fractal structures in their designs.

Theoretical Framework:

1. Fractals:

1.1. Definition of fractals:

The term “Fractal” refers to broken; i.e. fractal designs are not geometrically smooth but are defined by components on a hierarchy of different scales. Fractals can be either built with: accumulated accretions (patterns of ordered heterogenicity, spikes, granulations, hairiness), or instead, have gaps or holes (perforations, sieves, hierarchically ordered spacings). In either case, fractal structures depart from smoothness and uniformity by breaking geometrical linearity.

There are various irregular objects in nature e.g. the profiles of clouds are never circles or curves(fig1a); cross sections of mountains are never perfect triangles but irregular geometric shapes and so on. Only fractals can describe all these irregular objects (1),.

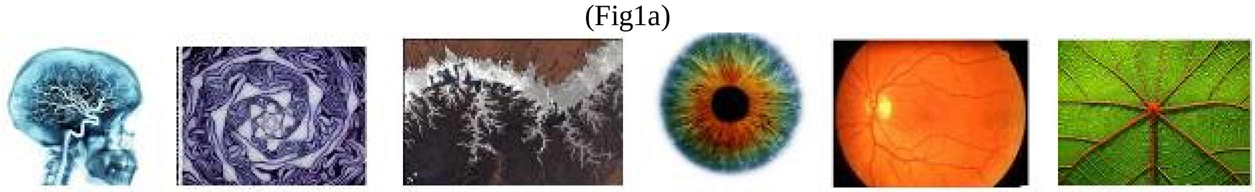
1.2. Types of fractals:

- **Natural fractal shapes:**

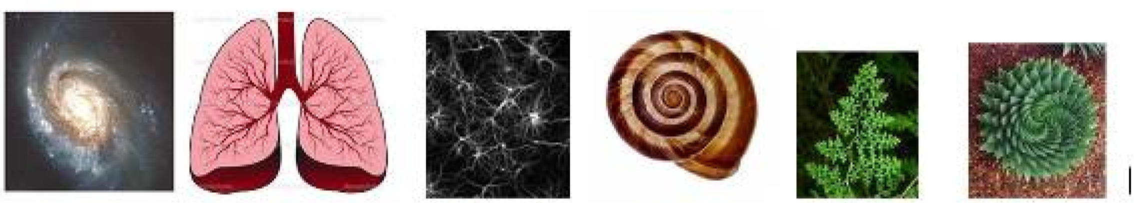
Natural fractals are widely found around us. Geometry and mathematical algorithms are hidden in those natural fractal shapes. Natural fractals come in the form of branching and spiral pattern (3) (4) (Fig1a, b).

Pattern of grass in the wind. Coastline, water winding in the mud (low tide) Sunrise at the Grand Canyon depicted like roots of a tree or axonal branches in neurons. the profiles of clouds .

(Fig1a)



'ractal branching of Blood Vessels in the brain.. Fractal spiral for cabbedge. Section Fractal branching of Lake Nasser, Egypt Fractal branching of Total eclipse of the eye Fractal branching inside the eye. Fractal branching of Vision of Leaves



Fractal spiral galaxy in the atmosphere. Fractal branching of Pulmonary artery. fractal branching from the human cortex. Fractal spiral shell. fractal branching of fern leaves Plant fractal spiral Fractal branching of lightning bolts

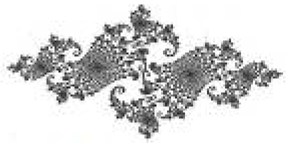
(Fig1b)

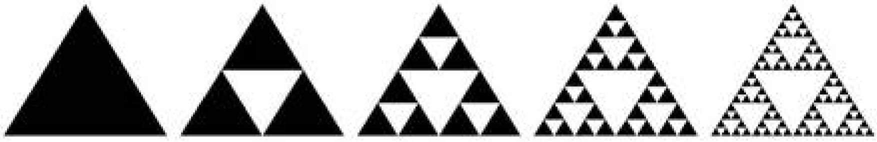
Natural fractal shapes.

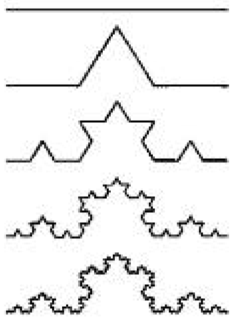
- **Geometrical fractal shapes:**

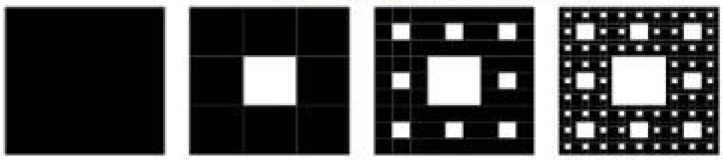
A geometrical fractal is a self-similar shape at every magnifying scale and whose fractal dimension exceeds its topological dimension, i.e. self-similar patterns recurring at progressively smaller scales. These self-similarities are not only the properties of fractals, but also, they may be used to define them and evaluate their dimensions (3). Geometric fractals can be made by repeating of parts that are in some way similar to the whole, such as Cantor's comb (Fig 2a), Gaston Julia sets (Fig 2b), Sierpinski's triangle (Fig2c), Sierpinski 's carpet (Fig 2d), and Koch's curves (Fig 2e) (5) .


a- Cantor's comb. (1872)(6)


b- Gaston Julia sets (1917) (6)


c- Sierpinski 's Triangle:(1915)



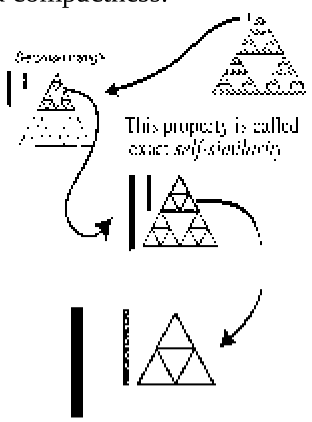

d- Sierpinski 's carpet

e- von Koch's curve (1904) (6)

(Fig 2a, b, c, d and e) geometric fractal shapes .

1.3. Fractal properties:

Fractal properties have: self-similarity, fractal dimension and compactness.



(Fig3) a fractal looks the same over all ranges of scale

- **Self-similarity:**

Mandelbrot defined the self-similarity as the following: "When each piece of a shape is geometrically similar to the whole, both the shape and the cascade that generate it are called "self-similar (1). "Similar" means that the relative proportions of the internal angles and shapes sides remain the same (Fig 1), (Fig2) and (Fig3).

There are three kinds of self-similarity:

- **Exact (perfect) self-similarity:**

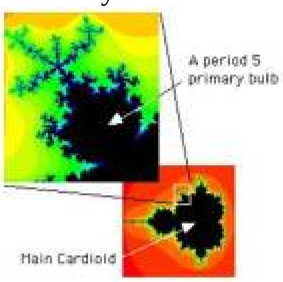
This is the strongest kind of self- similarity where the fractal is identical at different scales (3). Typical examples are Sierpinski's triangle and koch's snow flake (Fig2).

- **Approximate (not so perfect) self-similarity**

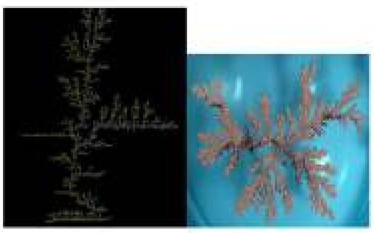
Approximate self-similarity is the less restrictive type of self-similarity. Fractals of this type contain distorted or degenerate copies of themselves. This type of self-similarity corresponds to most fractals found in nature (3) (Fig4a).

- **Numerical - Statistical self-similarity:**

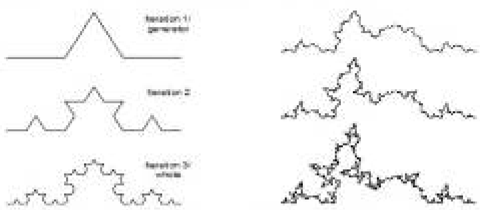
In these fractals, only the numerical and statistical properties are preserved across all scales. It is the weakest of the three (3) (Fig4b).



(Fig4a) Mandelbrot set and a magnification of a primary buls



(Fig4b) Computer generated Brownian tree made by diffusion limit .



(Fig4c) exact fractals vs statistical fractals.

- **Fractal dimension:**

The geometrical shapes have either regular shapes that can be studied by regular integer dimension: 1, 2 and 3 for line, surface (as square and circle) and volume (as cubic), respectively, or fractal irregular shapes that can be studied by non-integer fractal dimension having ranges between 1 and 2 & 2 and 3 (3), (5).

- **Compactness:**

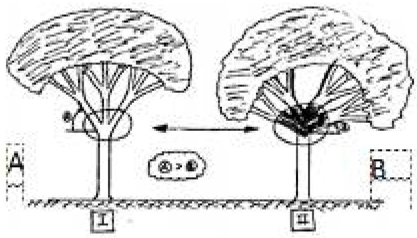
The fractal shape has infinite length, yet it encloses finite area (6) (Fig 1,2,3).

1.4. Properties of the fractal structure design:

1- The property of self-similar repetitions of fractal geometry shapes can be useful for designing structures whose members are replaced by sub-member and the sub-members are further replaced by their own sub-members, thus maintaining high- strength and saving huge amount of weight (6).

2- The feature of self -similar repetitions is a strategic outcome for functional, aesthetical, and environmental adaptation reasons. For examples: in nature, the self-similar branches of trees are the most common examples. The structural responsibility of branches is to spread the leaves to obtain maximum surface area for absorbing maximum sunlight during the whole day. The loads of leaves, fruits, and the branches, pass towards the main trunk. With the same concept and by fractal -like geometric configurations, designers can inspire their design (7).

3- Balance is also a characteristic of fractals structure design. For the two trees I, II, respectively: A and B are the two angles between horizontal line at the top of trunk and the nearest branche (fig5), M and N are the amount of material required at the circle junction of the trees, also m and n are the number of symmetric branches. To achieve balance for both trees and by comparison the conditions A>B and N>M also n > m are exist (8).



(Fig 5) Balance of fractals of both trees.

4- The property of the complex appearance of fractal shapes, which follows simple production rule is an excellent key tool that has potential to create innovative complex and intricate structures, or a radically new assemblage of structural members that contribute to offer unique structural behavior (9).

2. Generating fractal lattice "light-weight, high-strength" structure design:

2.1. Some simple mathematical definitions used with fractal:

In what follows we consider some simple and essential mathematic definitions used with fractal and must be well known to interior designer.

(I) Mathematic signs: union "U" and intersection "∩":

Ex: if S_0={1,2,3,4} and S_1={3 ,4,5,6 } then $S_0 \cap S_1$= {3,4} , while $S_0 \cup S_1$= {1,2,3,4,5,6}

$S_0 \cup S_1 = S_0+S_1$, and $S_0 \cap S_1$= line h

In general: $S_0 \cup S_1 \cup S_2 \ldots \cup S_p = \cup^{p}_{n=0} S_n$ and $S_0 \cap S_1 \cap S_2 \ldots \cap S_p = \cap^{p}_{n=0} S_n$ (10)

(Fig 6) Definition of mathematic signs: union "U" and intersection "∩"

Affine contract Transformation:

Affine contract transformation f is a linear transformation which is a combination of translation, rotation, contraction and reflection. The equations of the affine transformation is represented by:

$$f_i=\lambda_i \begin{bmatrix} \mu 1 & 0 \\ 0 & \mu 2 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$$

], i=1,2,..m, m≥1

where λ_i <1 are contract values ; μ_1, μ_2 are reflection signs ; θ is angle of rotation and δ_x ,δ_y are displacements of its transformation (6), (7).

(II) Evaluating Dimension of fractal shape (Hausdorff Dimension):

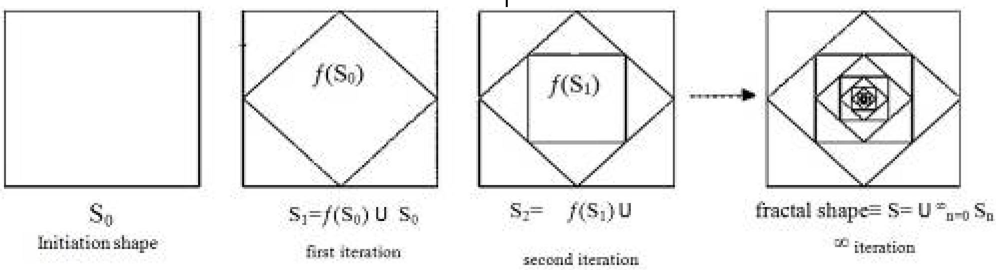
Hausdorff dimension $_D$ of any fractal shape is easy to evaluate by solving the equation, $\lambda_1^D + \lambda_2^D + \dots \lambda_m^D = 1$ ($\sum^m_{i=1}, \lambda_i^D = 1$), where $\lambda_i < 1$ (3) by using Newton Raphson iterative method (11). If $\lambda_i = \lambda$, for all i=1,2,..m then, $\lambda^D = 1/m \rightarrow {}_D = \log(1/m)/\log(\lambda)$.

(III) Iterative function system IFS method:

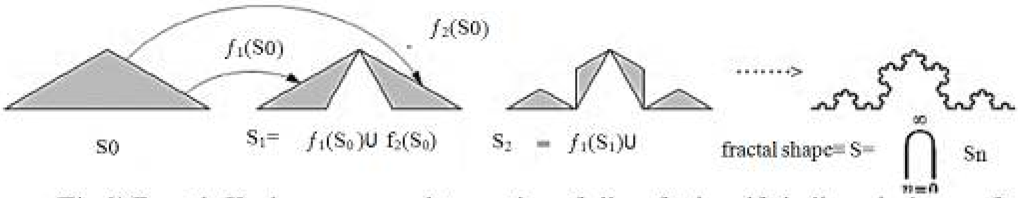
An iterative function system IFS method developed by V.F. Barnsley (12), to generate fractal which is self-similar. Hence the shape of a fractal is made up of several smaller copies of itself up to ∞ (3).

(IV) Construction of Geometric Fractal Shapes using (IV):

Using S_0, as initiation geometrical shape and after an infinite number of iterations where for each iteration, the finite number of affine contract transformation functions f_i, i=1,2,…m are applied, an infinite number of self -similar sub-shapes $S_1, S_2, \dots S_n, \dots$ each consist of a reduced copies of the original one are successfully achieved .The infinity state of S_1, S_2,..which is the required fractal shape S is resulted by either a **union** : $S = \cup^{\infty}_{n=0} S_n$ where $S_k = \cup^m_{i=1} f_i(S_{k-1}) \cup S_0$ (fig 7) or an **intersection** : $S = \cap^{\infty}_{n=0} S_n$, $S_k = \cup^m_{i=1} f_i(S_{k-1})$ (fig8), where k is an integer ≥1.(3)



(Fig 7) Fractals as a union of all perfectly self-similar sub-shape after affine transformation of f



(Fig 8) Fractals Koch curve, as an intersection of all perfectly self-similar sub shapes after affine transformation of f_1 and f_2.

(VI) Fractal lattice "light-weight, high-strength" Structure Design:

A fractal is an infinity entity. In practice, we replace infinity by finite positive integer p say, to end the iteration cycle. Therefore, only finitely iterated fractal models are applicable. Fractal lattice "Light-Weight, High-Strength" structure design is a geometric configuration constructed from all individual lines and their connecting points of fractal shape in $S \approx S_p$.

2.2. Four-steps iterative code to generate fractal lattice "light-weight, high-strength" structure design.

Manually, it is a hard process to design structures for fractal shapes since a high number of iterations are needed. Therefore, computer is the best solution to automatically design structures for fractal shapes. In what follows, we summarize the method of work in automatic four-steps simple compact computer code using IFS (iterative function solution) method, to generate from any geometric shape a fractal "light weight ,high - strength" geometric shape from which fractal lattice "light-weight, high-strength" structure design is constructed. Any designer with simple knowledge of fractal mathematic can use this code by calling it whenever needed.

Four- step computer code:

Input: S_0 initiation geometric filled shape; parameters $\lambda_i < 1$, i=1,..m ; μ_1, μ_2 ; θ ; δ_x, δ_y, and positive integer p>1 .

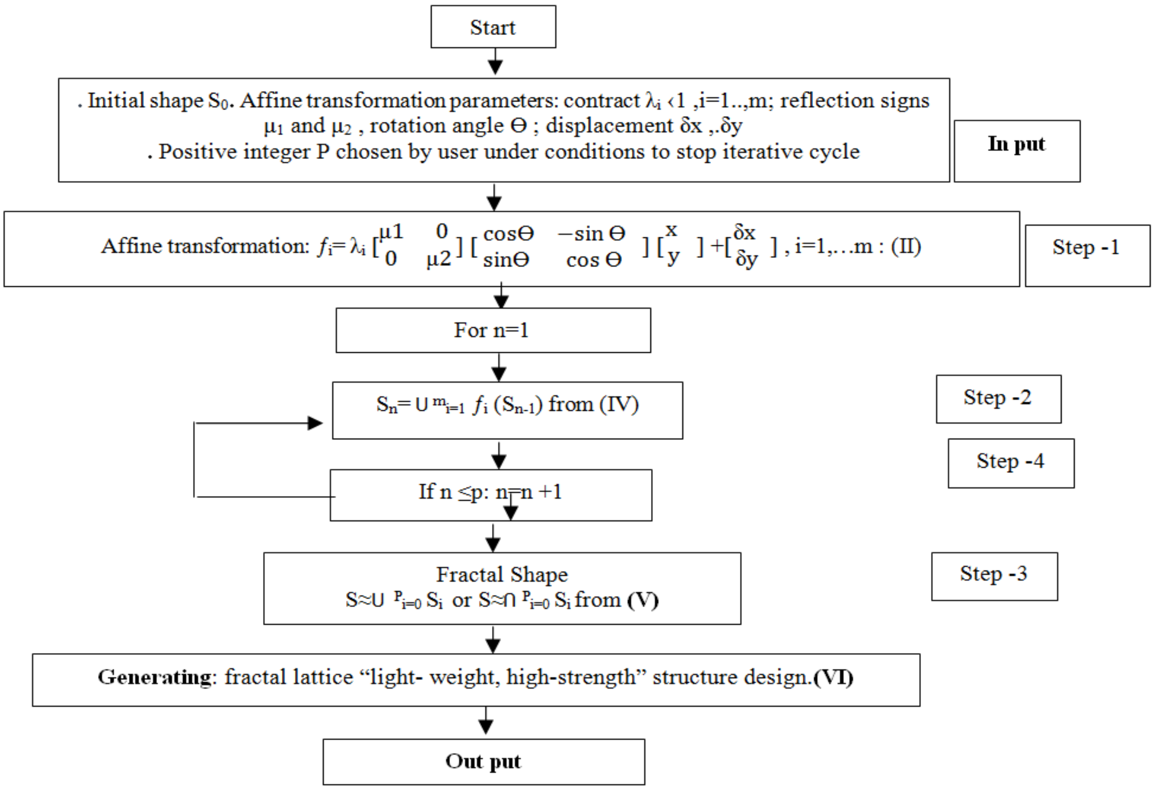
Step 1- Construct transformation functions f_i, i=1,.,m, using (II).

Step 2 - From S_0 Construct $S_1, S_2, \dots S_P$, $S_k = \cup^m_{i=1} f_i(S_{k-1})$.using (V)

Step 3- Construct "light weight-high strength" fractal shape S from either $S = \cup^p_{n=0} S_n$ or $S = \cap^p_{n=0} S_n$ using (V).

Step 4- Generate fractal lattice "light weight-high strength" structure design from S using (VI).

Flowchart for the four steps iterative code: Based on the previous four-steps iterative code description, each step can be scripted for computer automatically to easily generate fractal lattice "light-weight, high-strength" structure design as an output.



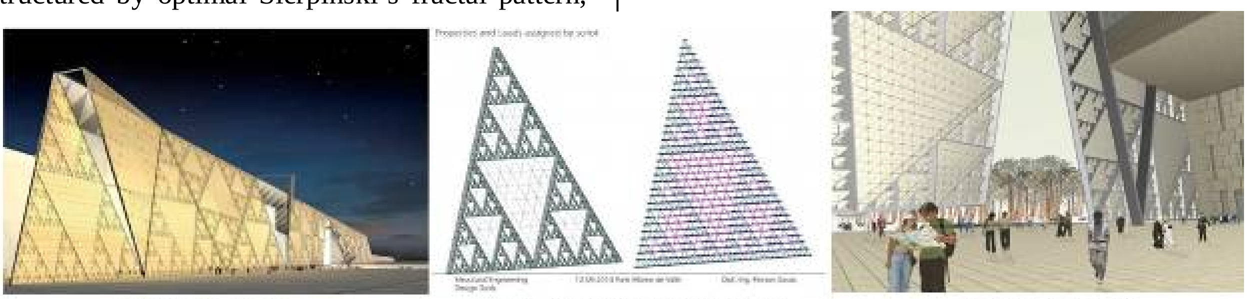
(Fig 9) flowchart for automatic 4-steps iterative code to generate fractal lattice "light-weight, high-strength" structure design.

3-Applications on fractal and fractal lattice "light-weight, high-strength" structure design:

a- Architecture:

Grand Museum of Egypt:

The facade surface is delicately fractured and structured by optimal Sierpinski's fractal pattern, simple in concept but creates structurally efficient optimal Sierpinski's fractal "light-weight, high-strength" system which is a very nice and pretty simple rational principle, creates a reciprocal grid adaptable to pyramid.

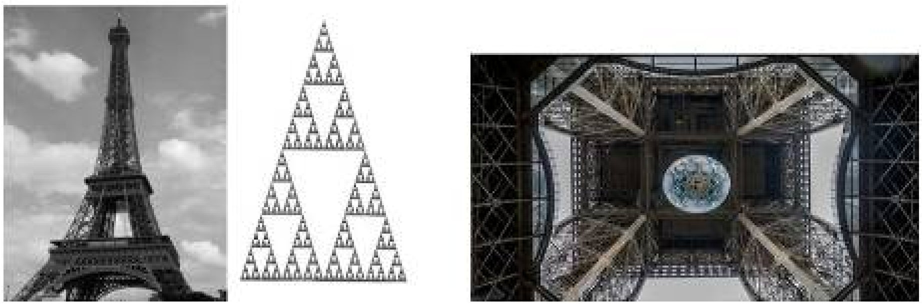


Façade exterior *Panel facade detail model* *Façade interior*

(Fig 10) Grand Museum of Egypt.

Eiffel's tower in Paris:

The Eiffel tower (1889) is an example of early use of fractal structural members. Rather than using big solid beams, they are made up of smaller beams in a self-similar kind of way with three levels of hierarchy**.**



(Fig 11) Gustave Eiffel's tower in Paris, where the repetition of a triangle generates a shape known amongst fractal geometrics as a Sierpinski's Gasket.

(Fig 12) Shopping mall in Addis Ababa, Ethiopia, designed by TED Fellow Xavier Vilalta , that design i based on fractal geometry.

(Fig 13) Institute du Monde Arabe, Paris, Jean Nouvel, that the design is based on fractal self-similar

(Fig 14) Museum, Illustration and Design Center, Aranguren & Gallegos Architects, Madrid, Spain, June, 2011

(Fig15)The Bird's Nest National stadium, china.2008

(Fig16) Mumbai internations Airport.

(Fig17)poly international palza

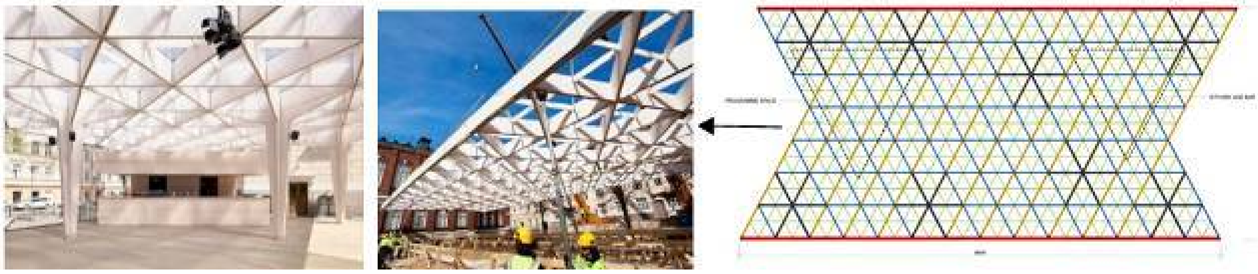
b- Interior design:

(Fig18)Studio in Moscow inspired by fractal geometric Sierpinski's triangle

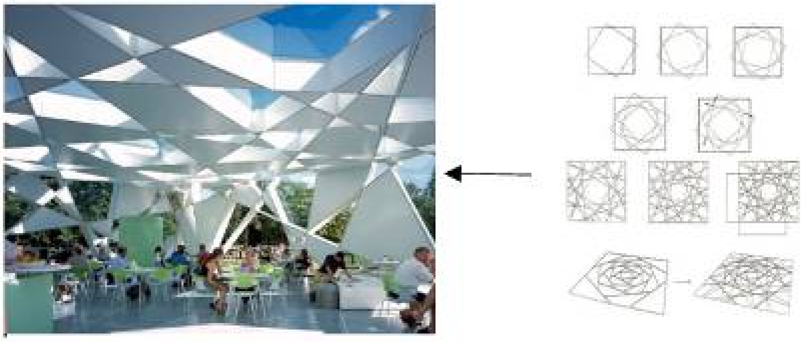
(Fig19) A fractal office design by perkins+will, august 2016, inspired by the fractal geometry can provide a contemporary, sleek office environment for businesses to thrive in.

(Fig 20)Munich fractal Arena by dear design: The structure's fractal design is based on infinitely repeated Xs. Each hole in the structure allows for easy access to the exhibited product, highlighted using white.

(Fig 21)Tang Palace Design by FCJZ



(Fig 22)World Design Capital Helsinki 2012 Pavilion, by Aalto University Wood Studio using the structure's fractal design Sierpinski's triangle lattice "light-weight, high-strength" in the ceiling.



(Fig 23) Serpentine gallery summer pavilion, 2002, London.

(Fig24)Different models of interior design based on lattice Sierpinski's triangle.

(Fig25) Partition inspired from fractal lattice "light-weight, high-strength" structure design

(Fig26) Shopping mall using form of branching & Sierpinski's triangle in ceiling.

(Fig27) Dynamic library design for London's Hyde park.

c-Outdoor design:

(Fig28)Designs inspired from lattice Nature fractal shape.

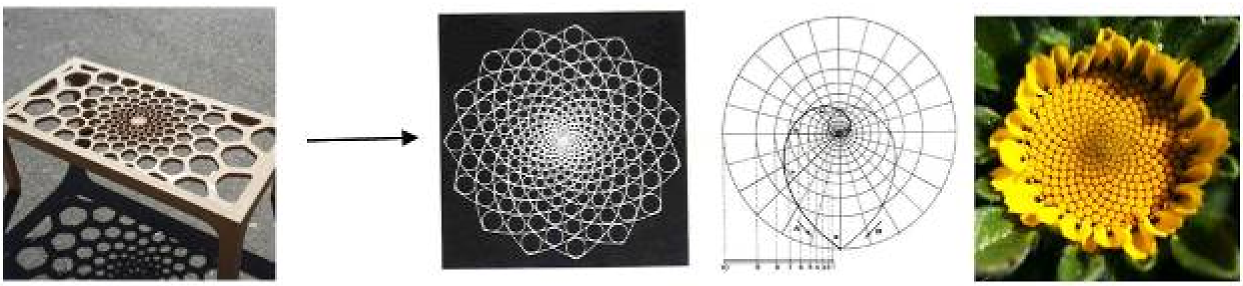
(Fig29) Nature fractal "form of branching" in architecture design of Singapore & Stuttgart Airport, Germany.

(Fig30) Designs inspired from lattice geometric fractal shape "square" structure design.

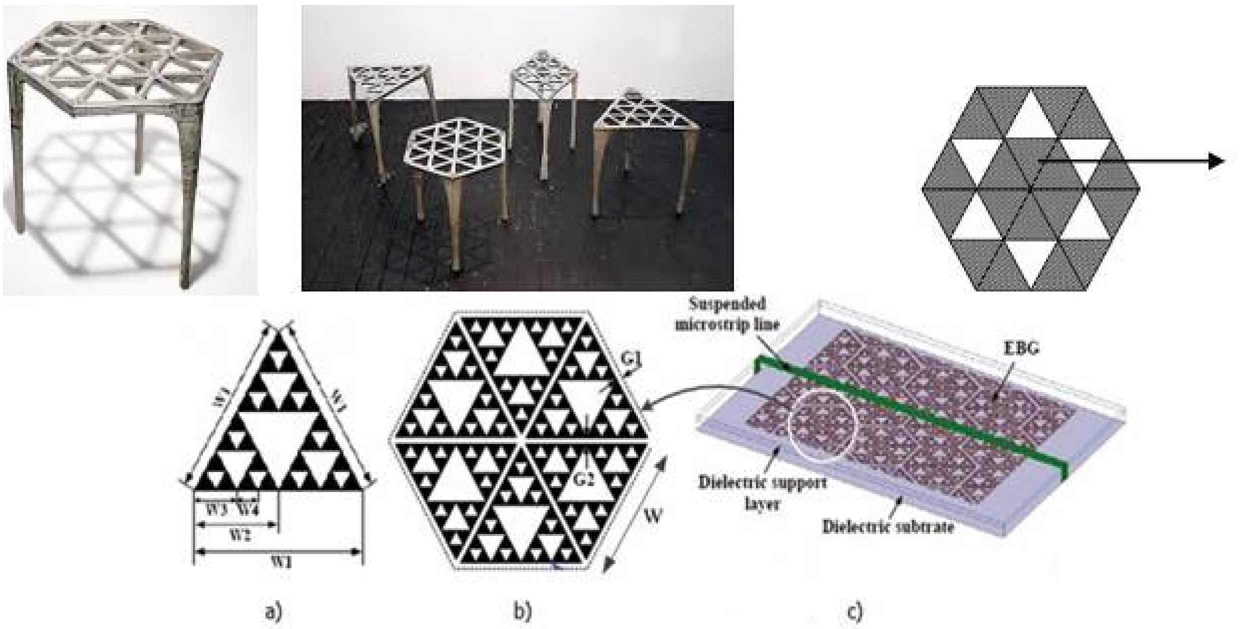
d-Furniture design:

(Fig31) Lattice Fractals self-similar for "square" in table and library design

(Fig 32) Applying Fractals self-similar lattice for pentagon and square in table.



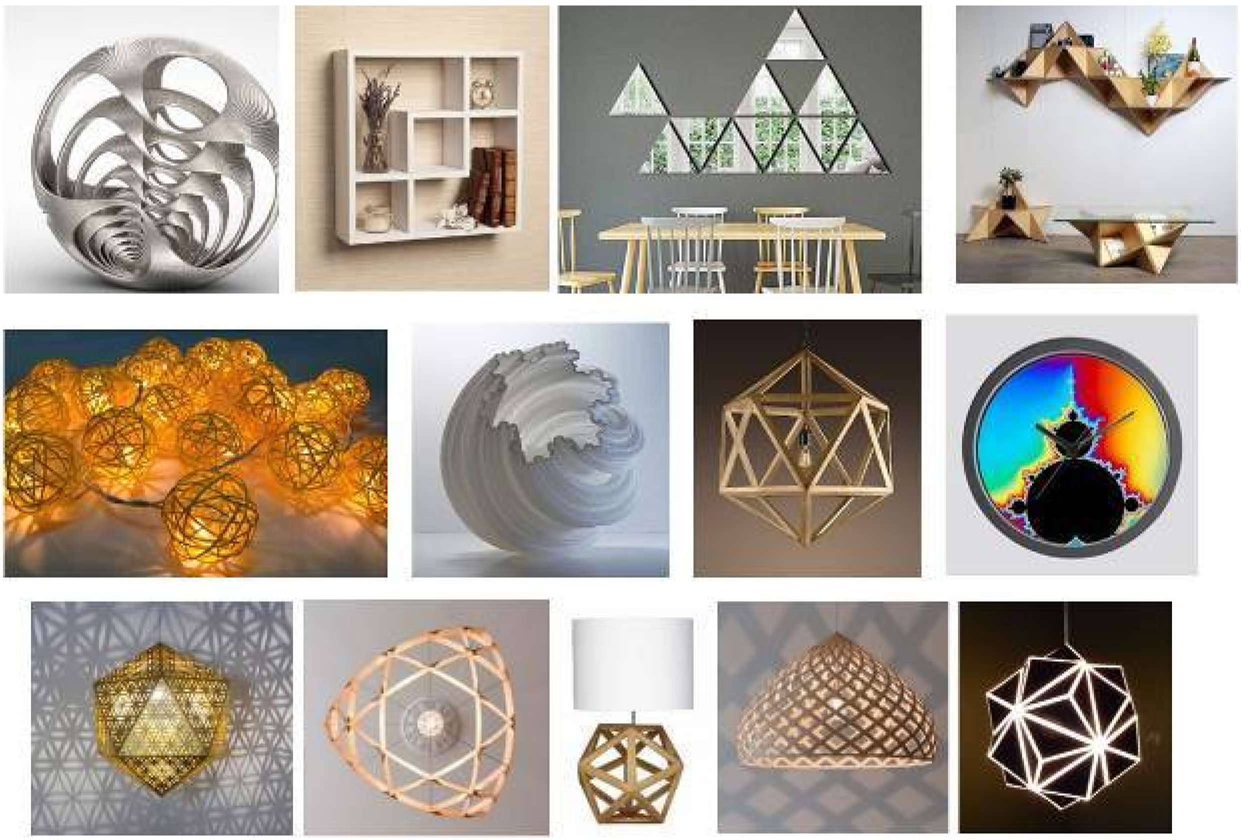
(Fig33) Table inspired from Nature Fractals applying concept of lattice.



(Fig 34)Different design of tables and partitions applying lattice Sierpinsky fractal "light-weight, high-strength" structure design.

(Fig 35) Different design by using lattice fractal (non-self-similar) structure design.

a- Accessories:



(Fig36) Different design of accessories inspired from different type of fractal.

4- Golden Fractal "light-weight, high-strength" structure design:

- **Golden ratio:**

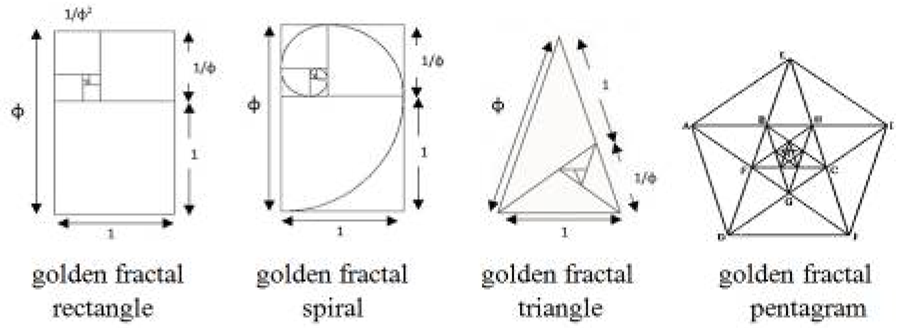
The golden ratio is $\varphi \approx 1.618$, where $1/\varphi \approx 0.618$ and $\varphi^n = \varphi^{n-1} + \varphi^{n-2}$.

- **Golden geometric shape:**

The golden geometric shape is a geometric shape whose side lengthes are related to the golden ratio φ, Ex: golden rectangle, golden triangle, golden spiral and golden pentagon (Fig37). Golden geometric shape has the specific characteristic of emitting positive energy (beauty energy, arranging energy) in the surrounding space. It plays an important role in protecting the space and the user from harmful effects by balancing the negative energy in the space; achieve health and high levels of human body comfort. (14)

- **Golden fractal shape:**

Golden fractal shape is generated from a golden geometric shape by iteration (Fig37)



(Fig37) golden fractal shapes from golden geometric shapes.

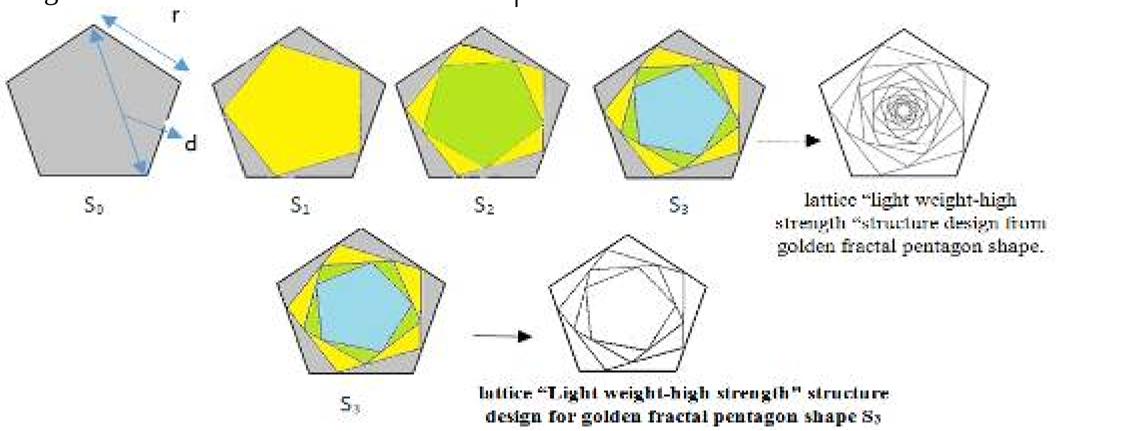
4-1 Golden fractal lattice "light-weight, high-strength" structure design:

In what follows, we show examples using the four-steps iterative code on different golden geometrical shapes to generate golden fractal shapes "light weight-high strength" then achieving golden fractal lattice "light weight-high strength" structure design models.

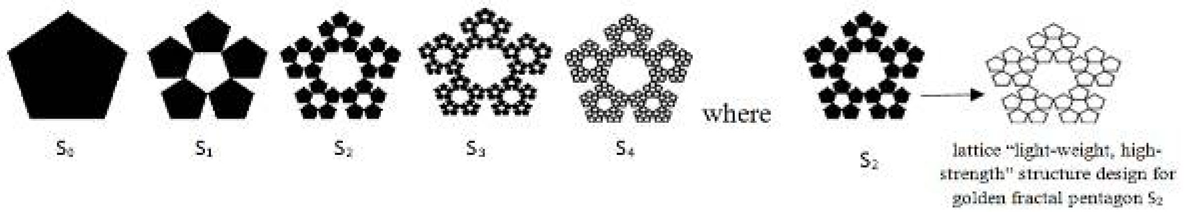
4-1-1 Golden fractal pentagon and pentagram Lattice "light-weight, high-strength" structure design:

(a) Golden fractal pentagon

The golden pentagon is a symmetrical 5 sides geometrical shape with relation d= φ .r, where d is the diagonal length and r is the side length (15)



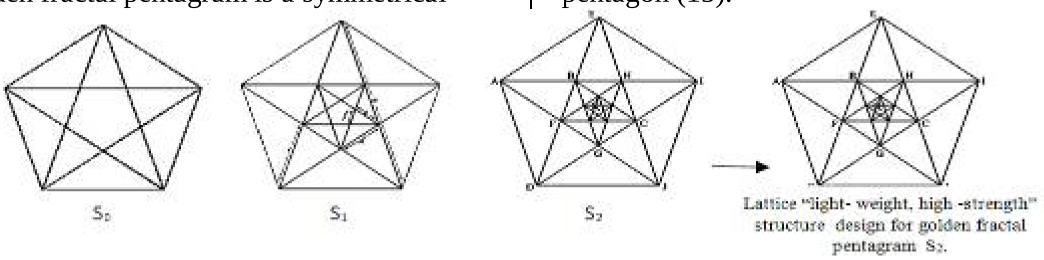
(Fig 37) we use Affine transformation; iterative sequence S_0, S_1, S_2, S_3; from S_3 as golden fractal pentagon shape "light-weight, high-strength" generate golden fractal pentagon lattice "light weight, high strength" structure design.



(Fig 38) we use Affine transformation; iterative sequence S_0, S_1, S_2, S_3, S_4; from S_2 as golden fractal pentagon shape "light-weight, high-strength" generate golden fractal pentagon lattice "light-weight, high-strength" structure design.

(b) Golden fractal pentagram:

The golden fractal pentagram is a symmetrical 5-pointed golden star that fits inside a golden pentagon (15).



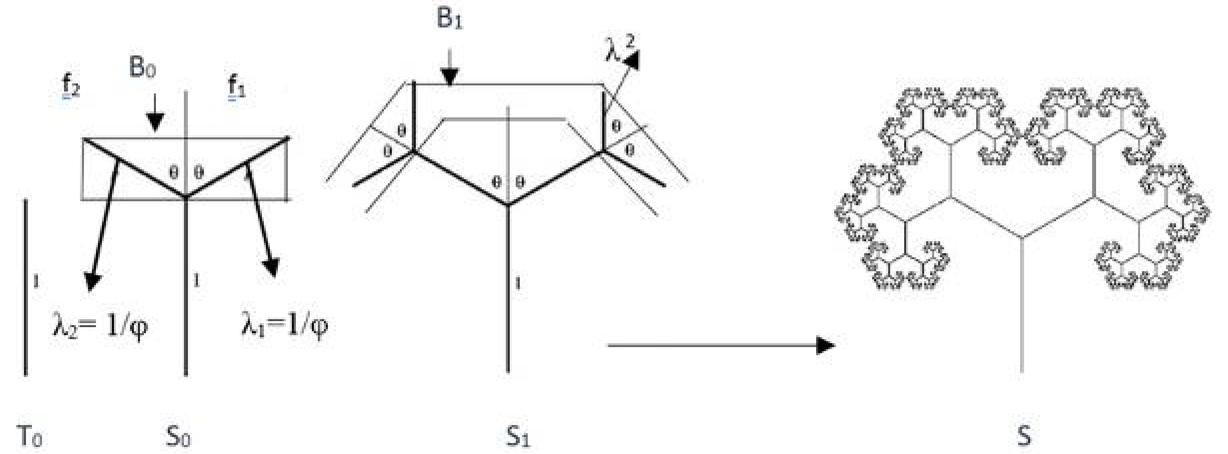
(Fig39) we use Affine transformation; iterative sequence S_0 ,S_1,S_2; from S_2 as a golden fractal pentagram shape" light-weight, high-strength" generate golden fractal pentagram lattice "light-weight, high-strength" structure design.

4-1-2 New Lattice Golden fractal binary tree "light weight – high strength" structure design:
Dr Yannick Joye of university of Gent said that "it is not the tree that causes the pleasing emotions for human beings but the fractal shape of the tree"(16). In what follows, three types of lattice "light weight-high strength" structure design for golden fractal binary trees are considered:

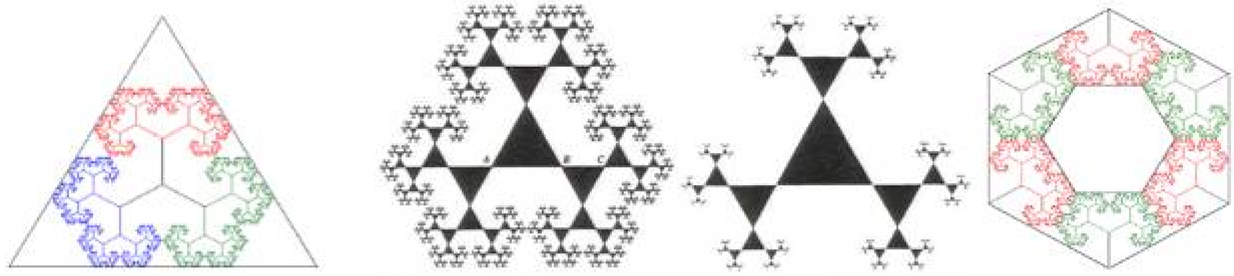
(a)Golden fractal self-symmetry binary tree with angle Ɵ=60
A fractal symmetric binary tree is constructed by choosing an angle Ɵ and a scaling factor λ, 0< λ <1, start with a vertical line segment T_0 (the trunk) of length unity. The trunk splits into two branches at the top such that each form an angle Ɵ with the linear extension of the trunk, one to the left and one to the right. Each branch has length λ and forms the trunk of a sub tree that splits into two more branching following the same rule, the angle is again Ɵ and the length of each of the four new branches is λ^2 and so on. The right and left affine transformation f_1 (T_0) and f_2 (T_0) are evaluated according to equation (II) of section 2.

$$f_1=\lambda\begin{bmatrix}\cos\Theta & \sin\Theta\\ -\sin\Theta & \cos\Theta\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix},\quad f_2=\lambda\begin{bmatrix}\cos\Theta & -\sin\Theta\\ \sin\Theta & \cos\Theta\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}.$$

$B_0=f_1(T_0)\cup f_2(T_0)$ and $S_0=B_0\cup T_0$ and so the first tree is born. Successively, $B_n=f_1(B_{n-1})\cup f_2(B_{n-1})$, n=1,2…., hence the group of trees are $S_n=S_{n-1}\cup B_n$ for n=1, 2, .. Choosing Ɵ=60 and $\lambda_i=1/\varphi$, i=1,2 we achieve a golden fractal contact binary tree. (fig40) (16), (17) with Dimension D: $\lambda_1=\lambda_2=1/\varphi\rightarrow 2(1/\varphi)^D=1\rightarrow D\log(1/\varphi)=\log(1/2)\rightarrow D\approx 1{,}44$ (using III)



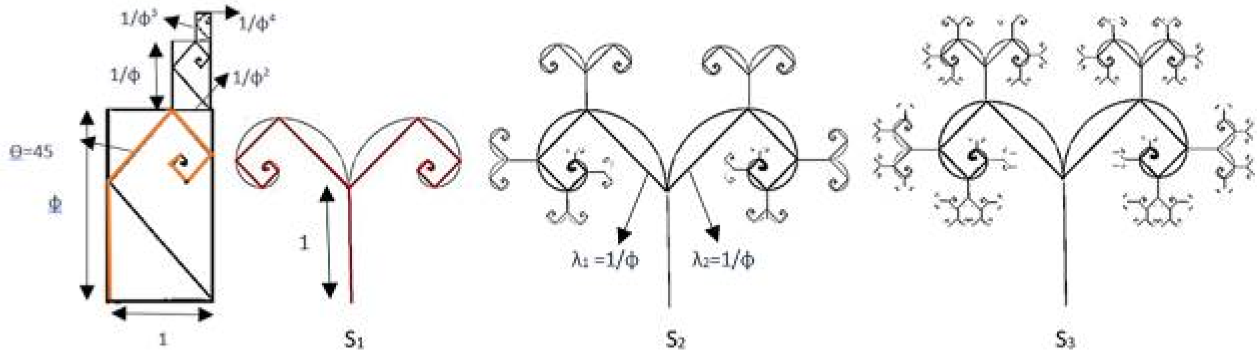
(Fig40) lattice "light weigh- high strength" structure design for self-symmetry golden fractal binary tree: using affine transformation, two branches are obtained from the trunk with scaling factor 1/φ, rotating counter clock wise by Ɵ (left) and - Ɵ (right), where Ɵ=60.



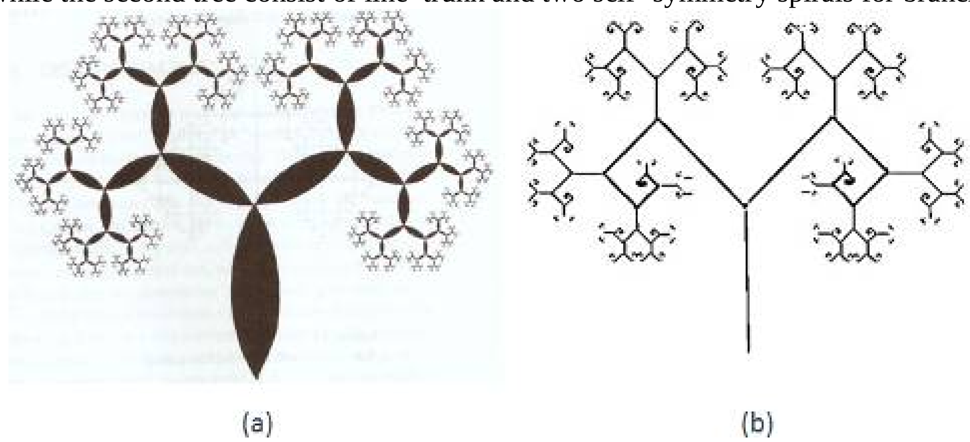
(Fig41) lattice Self-symmetry golden fractal binary trees as decoration models .

(b)New Golden fractal self-symmetry binary trees with Ɵ=45° and 90°(1rd model)
In what follows, the researcher construct two new styles of lattice "light-weight, high-strength" structure design for golden fractal self-symmetry binary trees, based on using the golden rectangle shape.

(b-1) New Golden fractal self-symmetry binary tree with angle Ɵ=45°
It is constructed from the propagation of two opposite right and left self-symmetry golden rectangles with sides having ratio length (φ:1), $(1/\varphi:1/\varphi^2)$, $(1/\varphi^3:1/\varphi^4)$ .. respectively. The contract scaling factor is $\lambda=1/\varphi^2$ and Ɵ=45. For n=1,2,3…the n^{th} trunk has length $T_n=1/\varphi^{2n-2}$, and accordingly the n^{th} broken line branch has successive sublines of ratio length: $\sqrt{2}/\varphi^{2n-1}$, $\sqrt{2}/\varphi^{2n}$, $\sqrt{2}/\varphi^{2n+1}$,..(i-e with ratio 1/φ:1: φ,..), n=1,2,3,…..(Fig 42).For any single broken line branch, the scaling factor is λ=1/ φ. To evaluate D: $\lambda_1=\lambda_2=1/\varphi$, $2(1/\varphi)^D=1$, $D=\log(1/2)/\log(1/\varphi)\approx 1{,}44$.



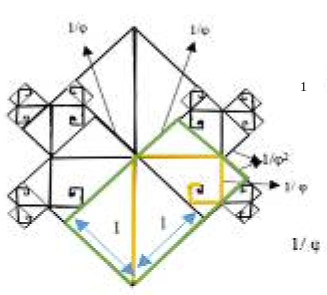
(Fig 42) lattice ‘light weight- high -strength’ structure design for self-symmetry golden fractal binary tree with Ɵ=45°. Its construction is based on two golden rectangles one to the right and one to the left. We have two styles of trees. The first tree consists of line trunk and two broken lines self- symmetry branches while the second tree consist of line trunk and two self -symmetry spirals for branches.



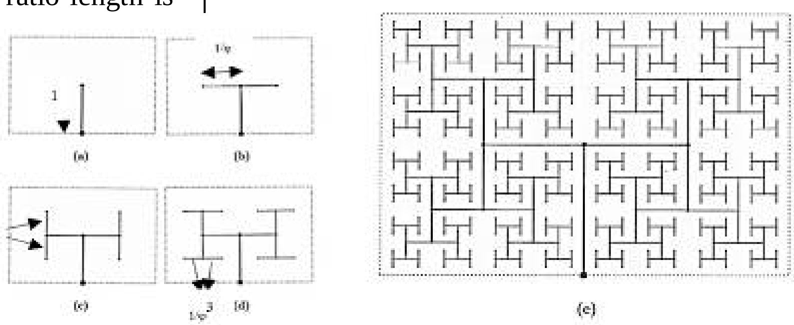
(Fig 43) Two lattice “light weight-high strength” structure design for Self-symmetry golden fractal binary tree with Ɵ=45° where trunk and branches are golden spiral for (a) and lines for (b)

(b-2) New Golden fractal self-symmetry binary tree with angle Ɵ=90°

It is constructed as a propagation for a combination of self -symmetry golden rectangles with sides ratio length (φ:1),(1:1/φ),(1/φ:1/φ^2), (1/φ^2:1/φ^3 …) . The scaling factor is λ=1/φ and Ɵ=90°(fig 44a,b). The nth trunk its ratio length is T_n= λ^n φ and its nth broken line branch has a successive sublines ratio length: λ^n,λ^n/ φ, λ^n/ φ^2,…, n=1,2,3,.. (i-e the ratio is : 1,1/φ,1/φ^2,..)for any single broken line branch,the scaling factor is λ=1/φ. To evaluate D: $λ_1$= $λ_2$=1/φ, 2(1/φ)^D=1, D=log(1/2)/ log(1/φ)=1,44.



(Fig 44a) lattice “light-weight, high-strength “structure design for self-symmetry golden fractal binary tree with Ɵ=90.

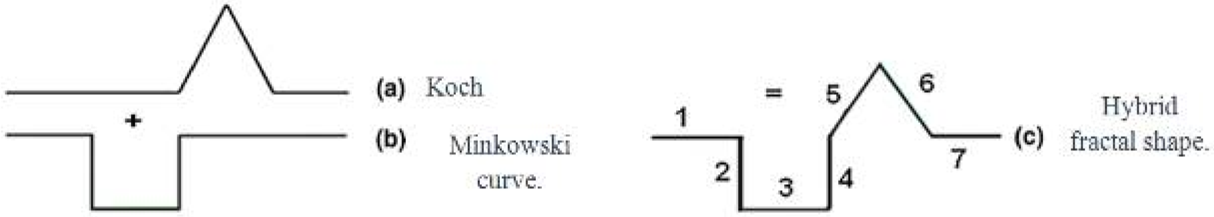


(Fig 44b) Construction area filled with lattice “light-weight, high-strength” structure design for golden fractal self-symmetry binary trees with Ɵ=90°.

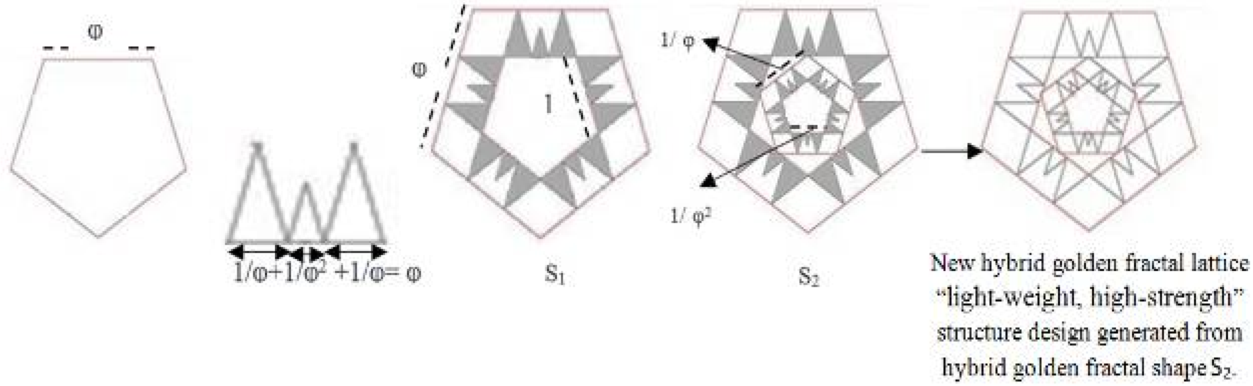
4-2 New golden fractal hybrid “light weight-high strength” structure design:
(2nd model)

Hybrid fractal shape is a two combined fractal shapes to form a new hybrid fractal shape which will be used as elementary shape generator to achieve further iterations of itself. Ex: Koch and Minkowski curve fractals.

Hybrid fractal



In what follows the researcher proposed a new hybrid golden fractal structure design based on using the golden ratio relation $\varphi^n=\varphi^{n-1}+\varphi^{n-2}$



(Fig45) Iterative sequence S_1,S_2 where S_2 is hybrid golden fractal shape "light-weight, high-strength" structure design

5- Optimal golden fractal "light-weight, high-strength" structure design:(3rd model)

(i)A mathematical perception for optimal fractal dimension:

The non-fractal dimension of the filled geometrical shape (plane) is D=2 and the non-fractal dimension of the line is D=1(fig 46). The interval 1<D<2 must be filled with fractal dimensions D=1.5+ϵ where -0,5< ϵ <0,5 for different types of fractal shapes. Perception studies of fractals generated by nature, mathematics and arts indicate that images in range D=1.3→1.5 have the highest aesthetic appeal (18), also gives the smallest rise in stress. The investigations included eye tracking ,visual preference, skin conductance and EEG measurement techniques(19). In what follows, by mathematical preception, the researcher proposed the existence of a mid-range interval with some preferable fractal dimensions D=1.5+ϵ,where ϵ is sufficently small positive or negative numbers [i-e the mid-range interval is (1.5+ϵ , 1.5-ϵ) where ϵ is sufficently small positive numbers] to obtain what we call the "best" or "optimal" fractal shapes . Dimension values outside this mid-interval should give a less preferable fractal dimension values.

(Fig46) comparison of patterns with different D values .

(ii)Geometrical analysis upon the perception of optimal fractal dimension:

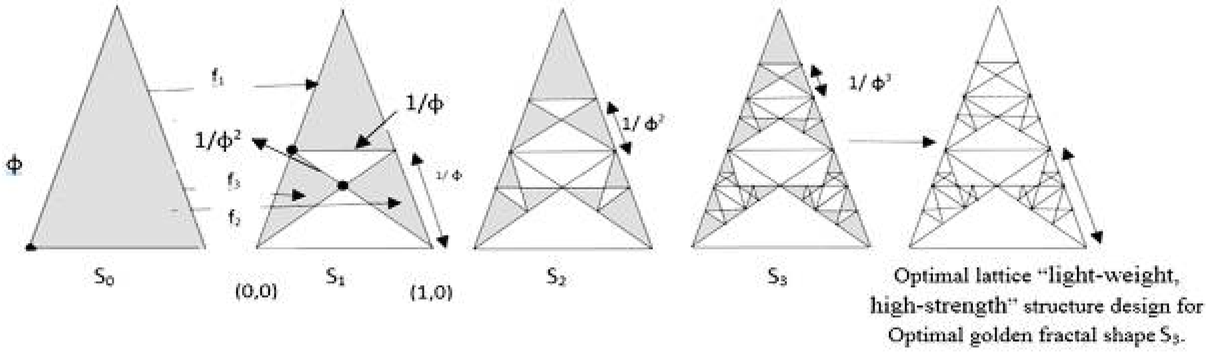
In what follows, the researcher considered a geometrical analysis to confirm the given mathematical perception. for optimal fractal dimension then to generate the optimal golden fractal "light weight-high strength" structure design. The Researcher proposed four filled golden triangle shapes (non-fractal D=2) as initial shapes. (The golden triangle is an isosceles triangle with two equal angles 72 ° and the apex angle 36 °, the ratio of the sides length of the triangle is 1: φ: φ). Apply the four-steps iterative code of section 2 on each one of the four filled golden triangle with different hierarechical self-similar transformation but related to each other by decreasing in the area of the punched holes(H) to generate four different "light-weight, high-strength" golden fractal shapes then four different lattice "light weight-high strength" structure design: cases a,b,c and d. To each case we calculate its parameters table to evaluate its dimension D and affine transformations f_i,i=1,2,…m from(III)and(II)of section 2.

Case (a): For m=3,

(Table 1)

IFS function f	Contraction λ	Reflection		Rotation Θ	Displacement	
		μ1	μ2		δx	δy
f_1	$\lambda_1=1/\varphi$	1	1	0	$(1/2\varphi^2)$	$(\sqrt{4\varphi^2-1})/(2\varphi^2)$
f_2	$\lambda_2=1/\varphi^2$	1	-1	36	1/2	$(\sqrt{4-\varphi^2})/(2\varphi)$
f_3	$\lambda_3=1/\varphi^2$	1	-1	-36	$(1/2\varphi^2)$	$(\sqrt{4\varphi^2-1})/(2\varphi^2)$

- To evaluate **D:** $\lambda_1=1/\varphi$, $\lambda_2=\lambda_3=1/\varphi^2$, $\rightarrow (1/\varphi)^D+2(1/\varphi^2)^D=1\rightarrow$ put $(1/\varphi)^D=x\rightarrow 2x^2+x-1=0$, $\rightarrow x=1/2\rightarrow D=\log(0,5)/\log(0,618)\approx 1,44=1,5-0.06$, hence optimal D= 1,44 and $\epsilon=-0,06$, with evaluated H=0.3925. (Fig 47)

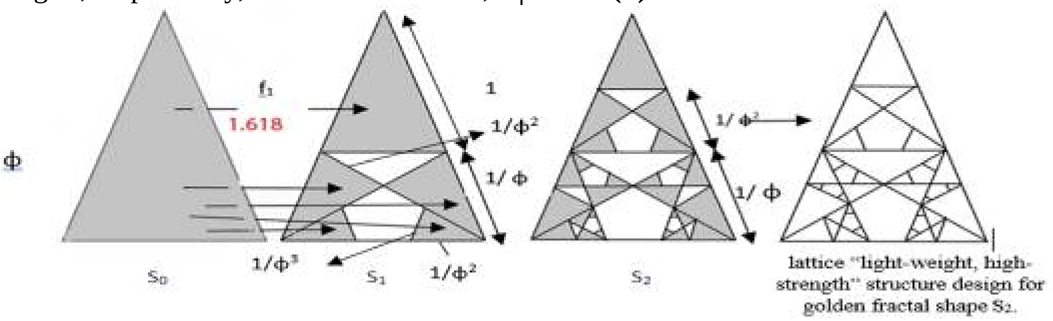


(Fig 47) we use affine transformation(f_1,f_2,f_3); iterative sequence S_0,S_1,S_2,S_3; S_3 is optimal golden fractal shape "light-weight, high-strength" structure design then generate optimal golden fractal lattice "light-weight, high-strength" structure design.

In the cases (b), (c) and (d), we tried to decrease successively the area of the punched holes by using a different but related transformation(changing the parameter values : contraction, reflection, rotation and displacement.)

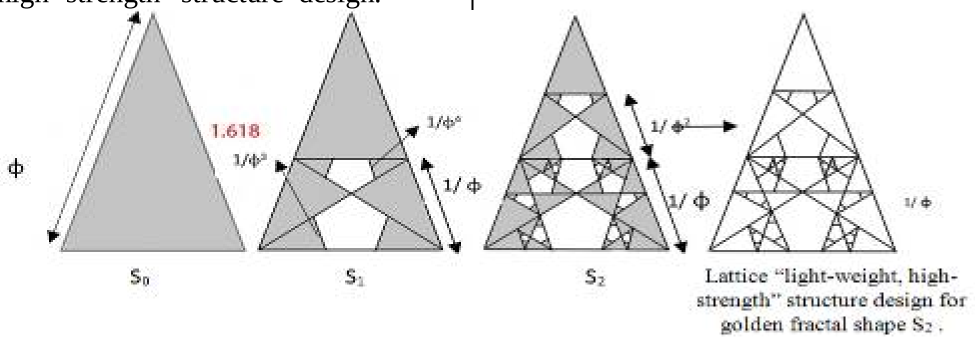
For cases **(b)** and **(c)** by repeating the process for golden triangles, respectively, with m=5 and m=7, $\lambda_1=1/\varphi$, $\lambda_2=\lambda_3=1/\varphi^2$, $\lambda_4=\lambda_5=1/\varphi^3$, $\lambda_6=\lambda_7=1/\varphi^4$, we achieved (fig48) and (fig 49) with D=1,703 =1.5+0.203 and D=1,785=1.5+0,285. Hence, ϵ=0,2032 and ϵ=0,285, respectively, which are not sufficiently small, so D is not optimal, evaluated H =0.307 and H=0.179, respectively .

Case(b) : m=5



(Fig 48) We use affine transformation(f_1,f_2,f_3,f_4,f_5); iterative sequence S_0, S_1, S_2 ; S_2 is golden fractal "light-weight, high-strength" structure design then generate golden fractal lattice "light-weight, high- strength" structure design.

Case(c) : m=7



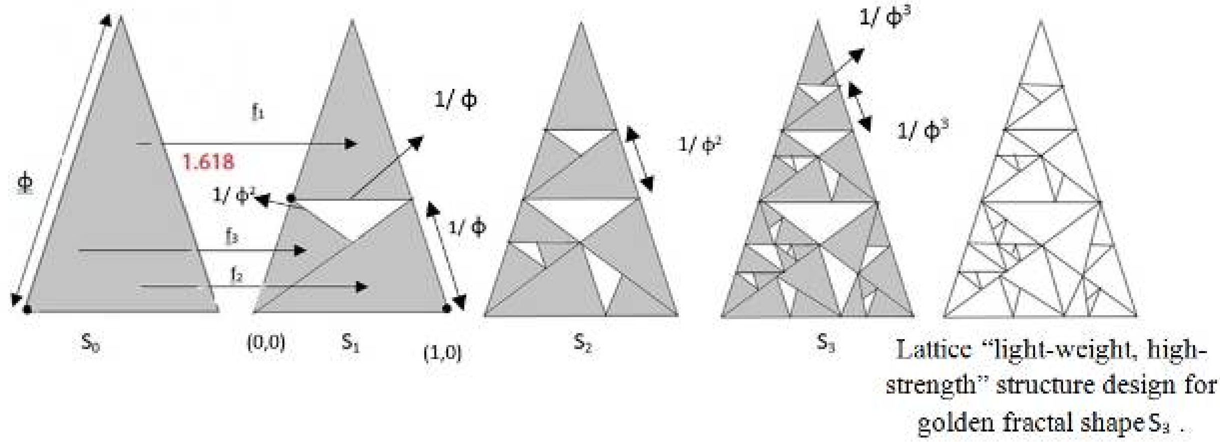
(Fig 49) we use affine transformation structure design($f_1,f_2,f_3,f_4,f_5,f_6,f_7$) ;iterative sequence S_0, S_1, S_2 ; S_2 is golden fractal shape "light-weight, high-strength" structure design then generate golden fractal lattice "light-weight, high-strength" structure design.

Case (d): m=3

(table2)

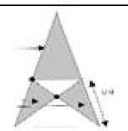
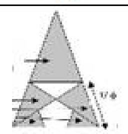
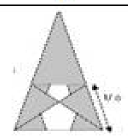
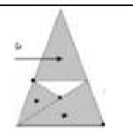
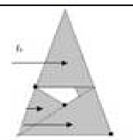
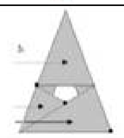
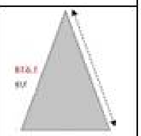
IFS function f	Contraction λ	Reflection μ1	μ2	Rotation Θ	Displacement δx	δy
f_1	$\lambda_1=1/\varphi$	1	1	0	$1/(2\varphi^2)$	$(\sqrt{4\varphi^2}-1)/(2\varphi^2)$
f_2	$\lambda_2=1/\varphi$	1	1	108	1	0
f_3	$\lambda_3=1/\varphi^2$	1	-1	-36	$1/(2\varphi^2)$	$(\sqrt{4\varphi^2}-1)/(2\varphi^2)$

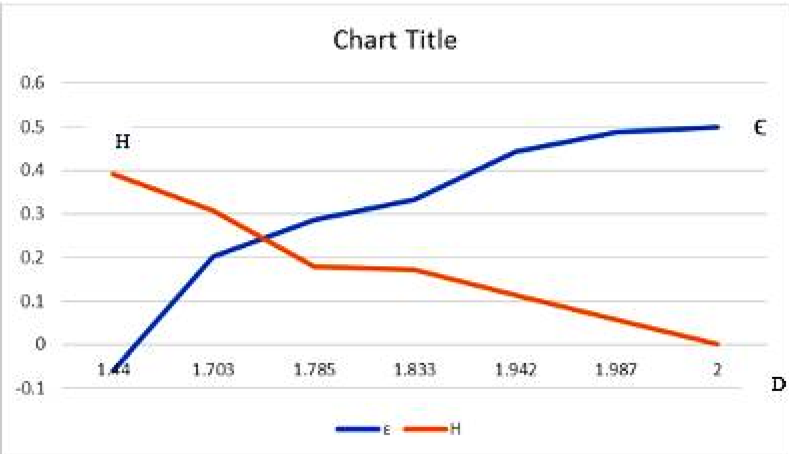
- To evaluate D: $\lambda_1=\lambda_2=1/\varphi$, $\lambda_3=1/\varphi^2$; then $2(1/\varphi)^D+(1/\varphi^2)^D=1$, for $x=(1/\varphi)^D$, $x^2+2x-1=0$, $x=-1+\sqrt{2}\approx 0{,}414 \rightarrow D=\log(0{,}414)/\log(0{,}618)$ $\approx 1{,}833=1.5+0.333$. Hence $\epsilon=0{,}333$ is not sufficiently small, so D is not optimal, evaluated H=0.172.



(Fig50) We use affine transformation(f_1, f_2, f_3); iterative sequence S_0, S_1, S_2, S_3; S_3 is golden fractal shape "light-weight, high-strength", then generate golden fractal lattice "light-weight, high-strength" structure design.

(Table 3) relation between dimension D and corresponding values of H(the values of holes area) and ϵ for different cases (a,b,c,d,e,f). case a gives the optimal fractal dimension D ≈1,44 with minimum value of ϵ and maximum value of H .

Cases	a	b	c	d	e	f	
S_1							
D	1.44	1.703	1.785	1.833	1.942	1.987	2
ϵ	-0.06	0.203	0.285	0.333	0.442	0.487	0.5
H	0,3925	0,307	0,179	0,172	0,114	0.056	0



(Graph1) shows similar increasing relation for both D and ϵ, also inverse relation between D(increasing) and H (decreasing) respectively.

From Table (3) and graph(1) we get the following results:

- Cases (a,b,c,d,e,f) show that there exists an inverse relationship between the fractal dimension D and H (the value of punched holes area in fractal shape). Also, a direct relationship between D and ϵ (the distance between 1.5 and D) exists . This means that, the optimal fractal dimension D has minimum value of ϵ and maximum value of H(minimum amount of material in constructing the fractal shape)
- Case (a) achieved the optimal fractal dimension $D\approx1.44=1.5+\epsilon$, where $\epsilon=-0.06$ is a negative sufficiently small value (minimum value), while evaluated H=0,3925 is the maxium value. The optimal golden fractal "light-weight, high-strength" structure design and its lattice form are generated in (fig47).
- To obtain optimal fractal shape, we need first to choose the optimal fractal dimension D by using the optimal self-similar regular hierarchcal transformation (contraction, reflection and displacement).
- In general, the optimal fractal shape "light weight-high strength" is the fractal shape with the dimension D lying in a particular mid-range interval ($1,5+\epsilon$, $1,5-\epsilon$) where ϵ is a positive sufficiently small number
- Not all fractals have the same degree of good fractal description, but specifically, optimal fractal shapes which have dimensions D lie in mid-range interval .These advantages are gradually lost outside this mid-range interval from both directions up to D=1 and D=2.

Case (v) :

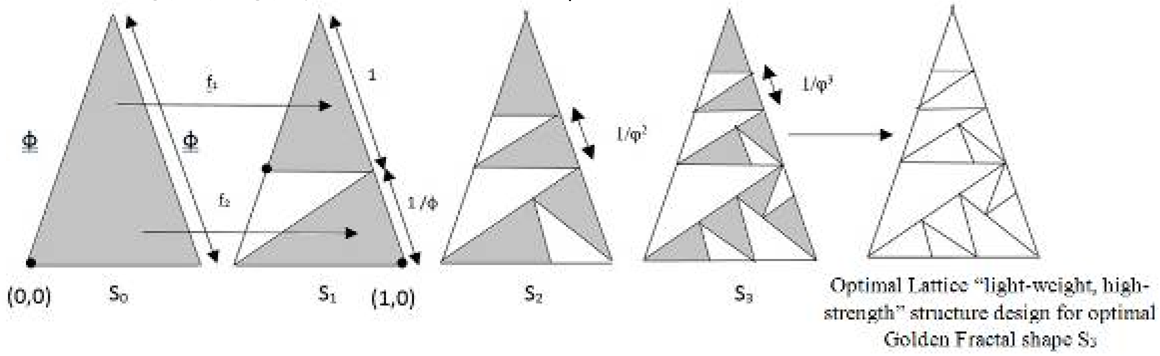
For m=2, using equations (II) sec.2, and table 4 then (fig 51) is generated

(table 4)

IFS function f	Contraction λ	Reflection $\mu1$	Reflection $\mu2$	Rotation Θ	Displacement δx	Displacement δy
f_1	$\lambda_1=1/\varphi$	1	1	0	$1/(2\varphi^2)$	$(\sqrt{4\varphi^2}-1)/(2\varphi^2)$
f_2	$\lambda_2=1/\varphi$	1	1	108	1	0

(table 4)

To evaluate D: $\lambda_2=\lambda_1=1/\varphi \rightarrow 2(1/\varphi)^D=1 \rightarrow D=\log(1/2)/\log(1/\varphi) \rightarrow D\approx 1,44$ where $\epsilon=-0,06$ and H=0,363



(fig 51) we use Affine transformation(f_1, f_2); iterative sequence S_0, S_1, S_2,S_3; S_3 is optimal golden fractal "light-weight, high-strength" structure design then generate optimal golden fractal lattice "light-weight, high-strength " structure design.

- Both cases(a and v) have the same optimal dimension $D\approx1,44$ for two different golden fractal shapes with two different values of H (Fig 51), (Fig47). That means their exist the same optimal fractal dimension for different optimal fractal shapes.

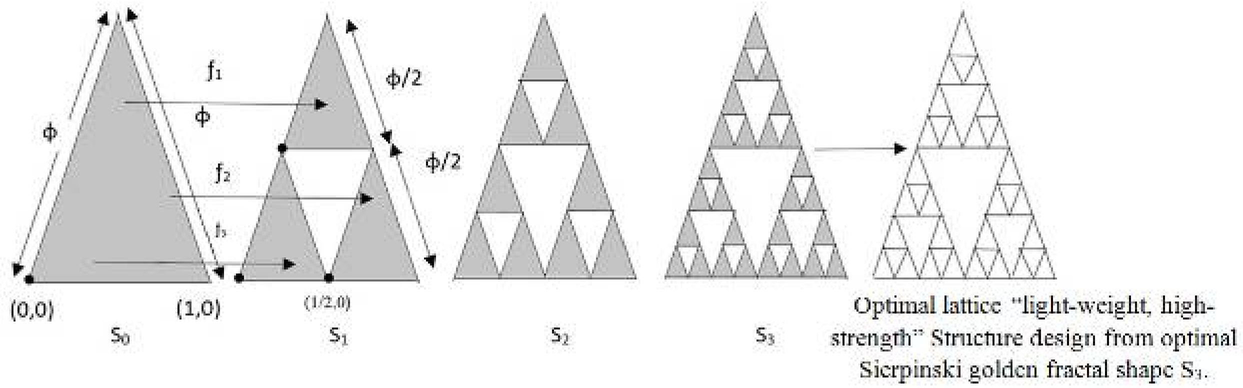
Case(w):

For m=3, using (II) section 2, and table (5), (Fig52) is generated

(Table 5)

IFS function f	Contraction λ	Reflection $\mu1$	Reflection $\mu2$	Rotation Θ	Displacement δx	Displacement δy
f_1	$\lambda_1=1/2$	1	1	0	¼	$(\sqrt{4\varphi^2}-1)/4$
f_2	$\lambda_2=1/2$	1	1	0	1/2	0
f_3	$\lambda_3=1/2$	1	1	0	0	0

$\lambda_1=\lambda_2=\lambda_3=1/2 \rightarrow 3(1/2)^D=1 \rightarrow D=\log(1/3)/\log(1/2) \rightarrow D=1,585$.where $\epsilon=0.085$



(Fig 52) we use Affine transformation(f_1,f_2,f_3,); iterative sequence S_0,S_1,S_2,S_3; S_3 is optimal Sierpinski's golden-fractal "light-weight, high-strength "structure design then generate optimal Sierpinski golden fractal lattice "light-weight, high-strength" structure design.

- D ≈ 1,585 is an optimal fractal dimension for optimal golden fractal Sierpinski's structure design since ϵ=0.085.

6- Using Golden fractal "Light-weight, high-strength" structure design to obtain a healthy and Sustainable interior environment.

The golden fractal shape is the language of nature, therefore, it plays an essential role in developing new forms of sustainable and healthy interior environment.

- **Health:**

- Inside the human body, the distributed electromagnetic charges keeps the body in balance. The body loses its balance when it is exposed to excessive electromagnetic charges. The golden fractal shapes act with these bad electromagnetic charges by emitting in the space an arranging energy which give balance to the space and to the human body [14].

- The human beings are apparently tuned to prefer an interior environment that has the self -similar properties which exist in fractals. This means that in any interior environment using fractal structure design, human body automatically dampens its response to stress [18].

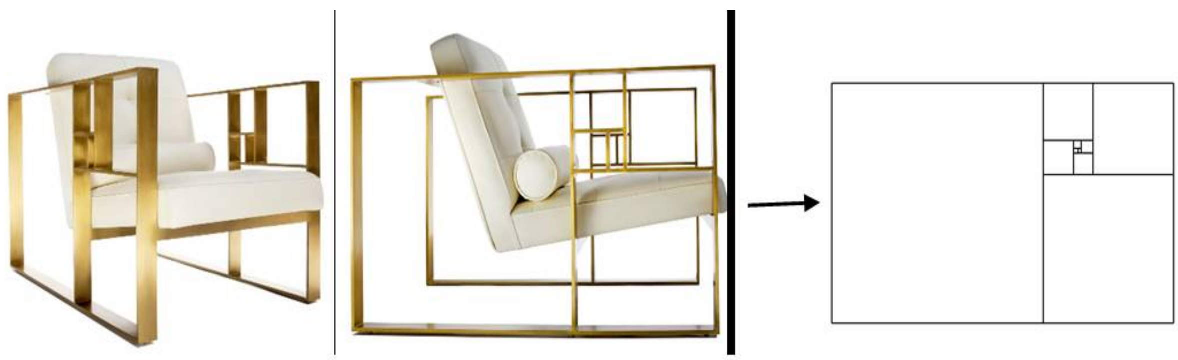
- **Sustainability**

As regards the sustainability using optimal golden fractal "light-weight, high-strength" structure designin interior design and furniture give the following:

- Bring aesthetical, harmony and balance to the interior space which are desirable.
- Maximizes the efficient use of interior space, since it minimizes the amount of construction materials and so relatively cheap. It is useful especially in small houses as it will serve as a solution in cities with quickly growing populations (ex. Egypt).
- Maximize the use of day light and minimize the use of artificial light with consequent reduction in energy through windows, doors, and roofs.
- Using environment- friendly material with small amount ,also recycled material of old furniture and decorative items to construct new optimal golden fractal "light-weight, high-strength" furniture, thus gives renovation to the interior space.
- Emits positive energy in the space to protect the occupant health from harmful effects and damps automatically the body response to stress, leading to more vital and more productive person.
- Converts the static interior space to dynamic, since it is flexible and easily moved and so it keeps the space active and attractive.
- stand the continuous and prolonged duration of functional usage.

7- Application of lattice Golden fractal "Light-weight, high-strength" structure design:

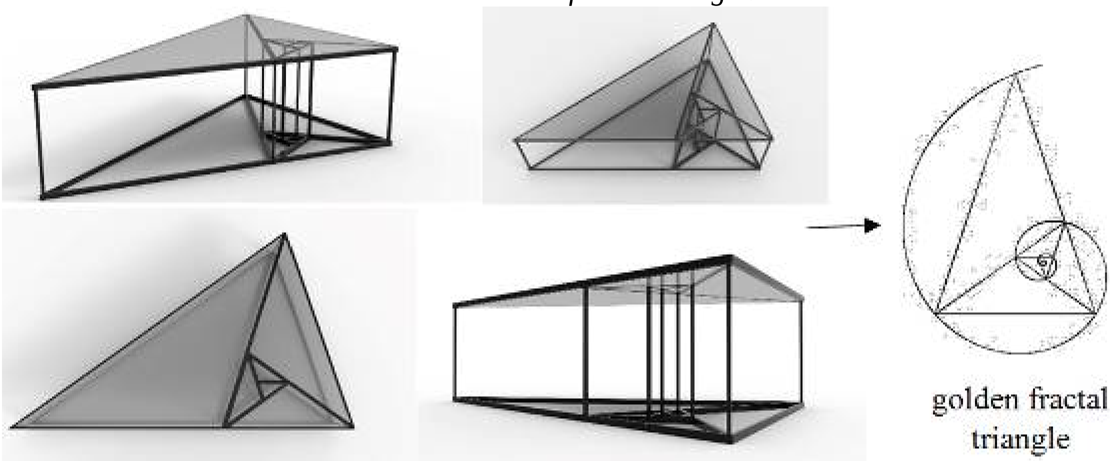
- *Golden fractal rectangle*



(Fig 53a) Different designs of furniture inspired from lattice Golden fractal rectangle .

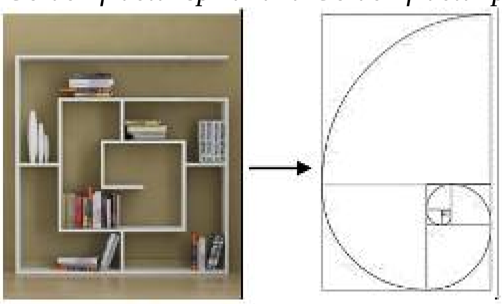
(Fig 53b) Different designs of furniture inspired from lattice Golden fractal rectangle .

- *Golden fractal triangle:*

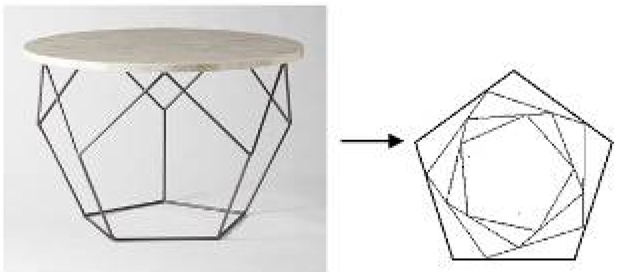


(Fig 54) Table inspired from lattice Golden fractal Triangle.

- *Golden fractal spiral and Golden fractal pentagon:*



(Fig 55) A library using Golden fractal spiral in design .



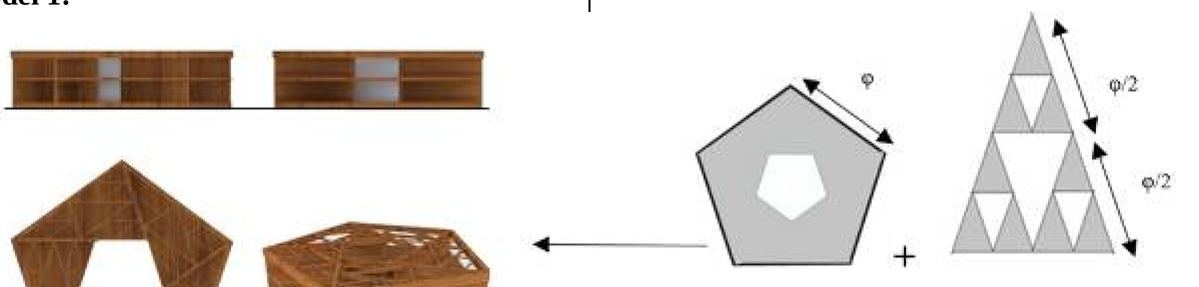
(Fig 56) A table inspired from lattice Golden fractal pentagon.

b- Suggested projects

All models used in this section are newly designed by the researcher.

Model 1:

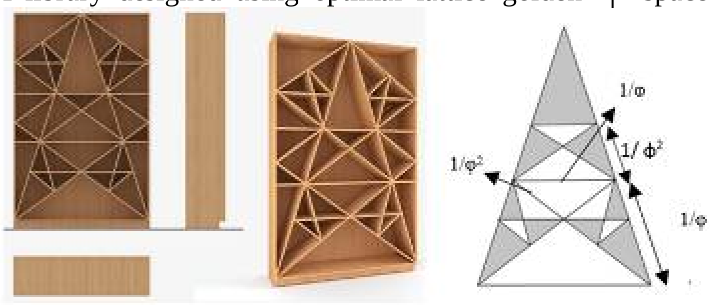
Two golden fractal shapes: pentagon and optimal Sierpinski's triangle, achieving a lattice "light-weight, high-strength" aesthetic table.



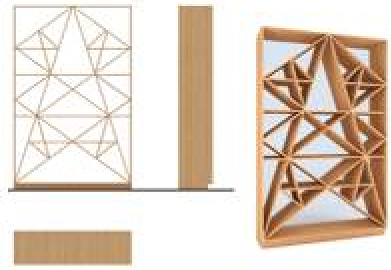
Hypride Golden fractal pentagon and Sierpinski's triangle "light weight- high strength" structure

(Fig 57) The design of this table depends on two golden fractal shapes: The pentagon and the optimal Sierpinski's triangle.

Model 2:
A library designed using optimal lattice golden fractal "light-weight, high-strength" structure design, emits positive energy around it in the space.



(Fig 58) The design of this library depends on using the new lattice optimal golden fractal shapes.

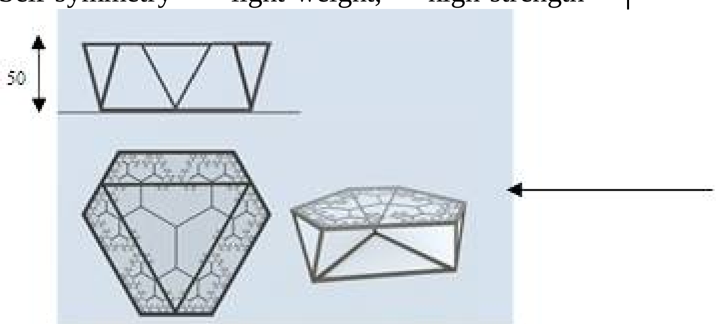


Optimal lattice "light-weight, high-strength" structure design for Optimal golden fractal shape

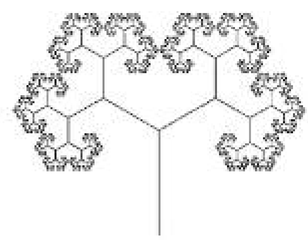
Model 3:
A banc with lattice golden fractal star "light-weight, high-strength" structure design.

(Fig 60) The lattice golden fractal star structure design used to design a simple banc

Model 4:
The design of this table is inspired from lattice Self-symmetry "light-weight, high-strength" structure design for golden fractal binary branching tree with ϴ=60.

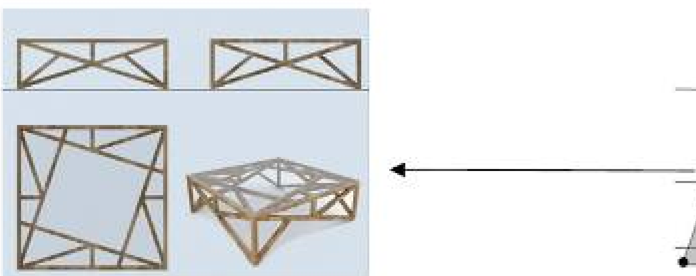


(Fig 61) The design of this table is inspired from lattice Self-symmetry structure design for golden fractal binary branching tree with ϴ=60.

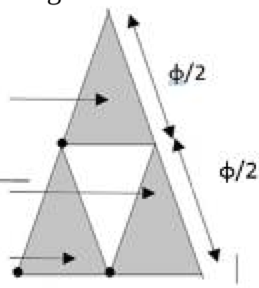


lattice "light weight- high strength" structure design for self-symmetry optimal golden fractal binary tree ϴ =60°

Model 5:
A simple table its design generated from lattice optimal golden fractal Sierpinski's triangle "light-weight, high-strength" structure design.



(Fig 62) The design of this table is inspired from lattice optimal golden fractal Sierpinski's triangle.



optimal golden fractal Sierpinski's triangle "light weight - high strength" structure design.

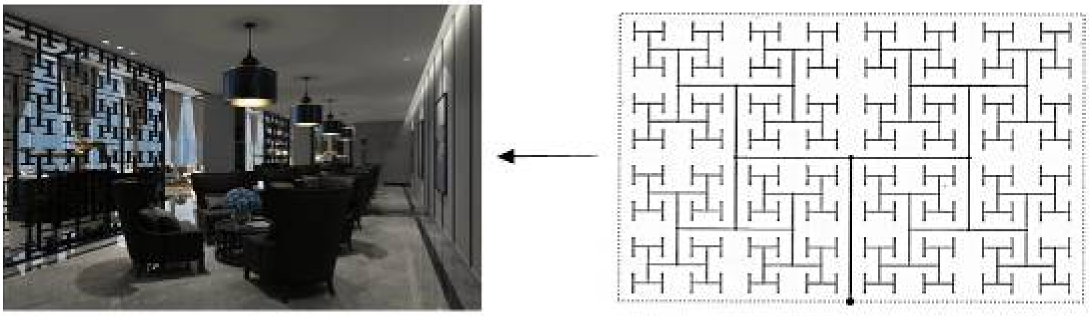
Model 6:

The design inspired from lattice “light-weight, high-strength” structure design for self-symmetry golden - fractal binary tree with Ө=45°.

(Fig 63) This design inspired from the new self-symmetry golden - fractal lattice binary tree with Ө=45°.

Model 7:

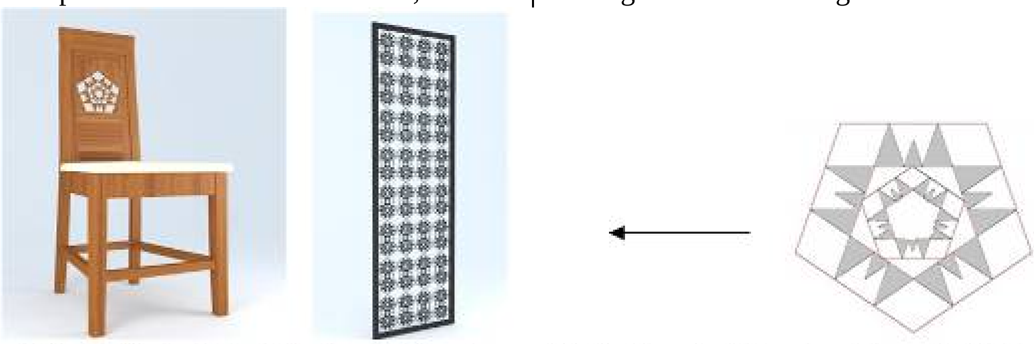
A simple partition between two interior spaces inspired from area filled with lattice “high-strength, light-weight” structure design for golden fractal self-symmetry binary tree with Ө=90°.



(Fig 64) we used the new optimal golden fractal shape to design a simple partition in any interior space.

Model 8:

The chair & partition have as a decoration, a new lattice hybrid golden fractal “light-weight, high-strength” structure design unit.



(Fig 65) The chair & partition inspired from the new hybrid golden fractal structure design unit which is obtained by the researcher.

Model 9:

Lattice” light-weight, high-strength” structure design for optimal golden fractal Sierpinski’s triangle used in designing simple partitions.

(Fig 66) The researcher designed a simple partition inspired from lattice optimal golden fractal ierpinski’s triangle to be used in any interior space.

8-Results & Conclusion:

1) The Golden fractal geometry is the essence of the geometry of nature, as it shows the way to understand the design of nature and shows the way to imitate nature designs. It is an essential foundation of interior design and furniture. It is a mathematical tool that reaches into the core of interior design composition to reach a different style of interior design and furniture that is adaptive, responsive, sustainable, and caring for human health.
2) Fractal geometry is based on hierarchical self-similar arrangement, which is an essential element to generate "light-weight, high-strength" fractal structure design by using the computer. From here, fractal geometry can be considered as an evolving concept in interior design and furniture.
3) The paper proposes the existence of a mid-range interval (1,5+ϵ ,1,5-ϵ) for optimal fractal dimensions D where ϵ is a positive sufficiently small number . The values of D outside this mid-range interval should give a less preferable fractal shapes. The fractal dimension D is a good indicator of a fractal shape.
4) The paper showed that the optimal fractal shape has the maximum value of punched holes- areas, with the minimum amount of material, which means reduces a huge amount of weight.
5) To construct optimal golden fractal "light weight-high strength" structure design, the interior designer must choose, first, the optimal self-similar hierarchical transformation with the optimal fractal dimension D, then apply the 4-step computer iterative code to obtain the optimal golden fractal shape.
6) This paper recommends that the interior designer must learn about natural mathematical rules, and its computer software to generate new fractal shapes and its structural designs.
7) The paper presents a simple computer method to transfer the golden geometric shape to lattice optimal golden fractal "light-weight, high-strength" structure design, which can be used in interior design and furniture,
8) The present work proposed new models of optimal golden fractal shapes, like self-symmetry binary trees with angles 90°,45°and others, which are obtained from golden geometric shapes, to be used in interior design and furniture .
9) Using Golden fractal geometric shapes in interior design and furniture bring aesthetic, harmony and balance in the space, changes the static interior space into dynamic and emits positive energy as well: (beauty energy, arranging energy and biological energy), enhancing the physical, mental, vital, emotional state and reduces stress for the occupant.

- The human beings recognize and respond positively to optimal fractal structure design because all organs and systems in the human body as the nervous system, the circulatory system and the lung system have similar optimal fractal shapes. This similarity links the human being cognitively with the structures that follow the same geometrical principles. the newly-generated innovative fractal interior design structure forms give inherently pleasant, healthy environment that reduces stress and activates brain areas associated with happiness to the occupant of the space. This similarity between the human body and the optimal fractal shapes provides a visual description for what is often an invisible, mysterious balancing act.

Recommendations:

- We recommend the usage of the fractal structure shapes in the interior design and furniture as it will help in the wellbeing of humans .
- We recommend that further studies should be performed with the collaboration between the interior designers & the medical physicians to detect the relationship between fractal shapes in interior and furniture design and its usage as a non-tradition form of therapy for different diseases.

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